

Mathematical Tool for Choosing the Best Material for Producing Masks

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Abstract: *There are specific rules for making professional face masks. These rules establish quality standards for masks that are used by medical professionals. The quality standards are based on the filtering power of bacteria and viruses, fluid resistance, breathability, etc. During periods of pandemic acute respiratory infections, the World Health Organization (WHO) use to recommend that professional masks should be a priority for medical professionals, for people who provide hospital and home medical assistance and for people with comorbidities. For other people, is recommended the use of non-professional masks. This type of mask generally is made from fabric and sold by artisans, or made by users themselves from the material that is within the reach. Unlike professional masks, there is no established norm or standard for the assessment of non-professional masks. Many studies have been carried out in order to measure the filtering efficiency of viruses, bacteria and the breathability of various materials for homemade masks, yet, no conclusions have been made on which type of mask is the best. We use an Operational Research approach to solve the problem of choosing the best mask to be produced from a set of masks made from different materials. We propose a mathematical optimization model which is formulated using the Extended Goal Programming. Computational experiments were conducted to analyze the trade-off between different performance measures of materials in the selection of the best mask. This model serves as a very useful tool to be used in the selection of the type of non-professional mask to be made.*

Keywords: Cloth mask, Pandemic, Filtering Efficiency Measures, Multicriteria Optimization, Goal Programming

1. Introduction

In the recent months, the whole world has been looking for measures that can help control the transmission of novel coronavirus (SARS-CoV-2). One among several of these measures is the use of a mask, [21].

The use of surgical mask or N95 respirator (degree of protection certified by the National Institute for Occupational Safety and Health, an FFP2 or an equivalent) is one of the measures that can help in the prevention of some viral respiratory diseases, i.e. they can prevent the spread of infectious droplets from an infected person to an uninfected person and also contamination of the environment, [3], [7], [15], [22]. Therefore, WHO (World Health Organization) encourages the use of a mask by the entire population. However the emphasize is that it is essential that medical masks are prioritized for health professionals, people who provide hospital and home medical assistance and for people with comorbidities. For other people, the recommendation is to use nonprofessional masks as a protective factor against contaminated respiratory droplets, [21].

The performance of the respirators and surgical masks is tested using a well defined set of international norms, including bacterial filtering efficiency (BEF), particulate filtering efficiency (PFE), fluid resistance, breathability, flammability, etc. On the other hand, there is no

standardization for nonprofessional masks. In this context, there are many studies which investigate the effectiveness and efficiency of the masks from homemade materials, [1], [6], [14].

Aiello et al. [1] studied the influence of use of face masks and hand hygiene in the reduction of the incidence of influenza-like illness. Students in a university residence halls were randomly split into 3 groups and observed in within a period of 6 months during the 2006–2007 influenza: students who were using face mask, students who combined face masks with hand hygiene and a control group. The result suggested that face masks and hand hygiene might reduce respiratory illnesses in shared environments and reduce the impact of pandemics.

Ten years later, Konda et al. [14] observed that little is known about the performance of available fabrics for non-professional masks. These authors evaluated filtering efficiencies as a function of aerosol particulate sizes (< 300nm and > 300nm) for several fabrics commonly used for making non-professional masks, including the combinations of fabrics and different quantity of layers. The authors concluded that the efficiencies improved when multiple layers were used, mainly when a specific combination of different fabrics were used, which contributed to the mechanical and electrostatic-based filtering. These authors showed also that, when the mask was not adjusted properly to the face, the filtering efficiency could decrease over 60%. They found that cotton, natural silk, and chiffon with a tight

weave, could provide a filtering efficiency above 50% in the 10nm to 6.0um range. Also the higher threads per inch cotton (for instance 600 TPI cotton) resulted in better filtering efficiencies. The final conclusion was that cloth masks had potentials to protect against the transmission of particles in the aerosol size range. In this work, the breathability of each type of fabric was also tested. However, little evidence was given to this important measure of performance.

Howard et al. [9] provided a review of research works, which studied the effectiveness of homemade materials for manufactured masks. The studied materials included silk, chiffon, linen, polyester, flannel, cotton T-shirt, scarf, tea towel, pillowcase, antimicrobial pillowcase, vacuum cleaner bag, paper coffee filter, and many others. Davies et al. [6] and Rossetitit et al. [16] also presented important results about effectiveness of materials for making homemade masks.

Although all these refereed works were concerned with obtaining parameter values that measure the effectiveness of different types of materials for making masks, there was no conclusion which of these materials was the best. Such a conclusion is indeed very difficult to make, since the measures of the performance of materials are conflicting. For example, the increase in the number of threads in the fabric or layers of the mask results in the improvement of the efficiency of particle retention, but consequently leads to the worsening of the breathability of the mask. In this context, given a set of possible ways of making a mask (with different materials and layers), this work proposes a mathematical tool to help choosing the best one, based on an optimization model. This model was formulated using Extended Goal Programming techniques.

2. Use of Masks

The mask is used to prevent the spread of infectious droplets from an infected person to an uninfected person and also to prevent the contamination of the environment. Therefore, during the outbreak peak phase of respiratory diseases, with a high risk of transmission, it must be used by all people, except children and adults who are not able to remove their own masks, [16], [2]. In case of scarcity, professional masks must be prioritized for people who provide healthcare service, people more vulnerable to becoming severely ill and infected people. For other people, it is recommended to use non-professional masks, which can be purchased from artisans or made by the user themselves with a homemade material, [21].

Masks require a lot of care, from their acquisition up to their use. When inappropriately used and hygienized, the mask can be an object of self-contamination. When purchasing a mask, the user should be attentive to the shape and fit of the mask on the face and especially of the material that the mask was made from. Konda et al. [14] showed that when there were leakages around the mask area, its particle filtering efficiency could degrade over 50%. These and many other authors have investigated the efficacy of the mask based on the materials that it is made from and concluded that there was a wide discrepancy in effectiveness between them, [6],

[16],[20].

3. Efficacy of Masks

The performance of a mask is closely linked to the material it was made from, shape and how well it fits the face. Professional and non-professional masks can be made from different materials and therefore have different. Performance as a protection barrier against contaminated respiratory droplets. When the mask is not adjusted, this protection can be reduced, [14]. Another very important factor is its cost. The more expensive, the less accessible to people with low purchasing power.

Medical (professional) masks are used only once and then discarded. On the other hand, fabric masks can be washed several times, and this number depends on the fiber structure of the fabric. Its durability also depends on the way that it is washed and the use of chemicals products (such as chlorine) or heating, [21], [16]. Studies to show how much a mask loses performance when washed and, consequently, the maximum number of times that each fabric can be washed without losing much of its filtering capacity, are still lacking. The durability of the mask and its cost can be compared using the cost-benefit analysis. Material strength and thermal comfort are also important information to obtain when purchasing a mask. The mask should not tear during use and also heating or cooling of the mask can cause a lot of discomfort to the user.

According to [13] the total comfort is the sum of thermal comfort, moisture vapor transport and aesthetic comfort, the thermal comfort being the most important. The thermal comfort is characterized by the temperature balance between the body and the ambient, i.e. the heat is eliminating or receiving from the body through the skin. It can also be influenced by personal factors such as sex, age, diet (metabolic heat flow), sleep pattern, etc. The mask has a direct contact with the skin and can make the removal of heat from the face difficult, interfering with convective heat transfer, as it prevents the movement of air close to the skin. The permeability of the mask-making materials and skin contact area can be obstacle to sweat, evaporation and make its use very uncomfortable, [17],[19].

The materials used for medical face masks have a standard specification for performance. In general, the performance is assessed by the standardized efficiency measures (ASTM F2100 EN 14683 European standard, EN 143/149, GB2626-2006, ISO 16900, etc.) including bacterial filtering efficiency (BFE), particulate filtering efficiency (PFE), fluid resistance, breathability, flammability and skin sensitivity and cytotoxic.

Viral and bacterial filtering efficiency tests are performed by connecting the mask to a breathing circuit filter. In the analysis of bacterial filtering efficiency (BEF), aerosols of bacteria are thrown through the mask and the filtering is observed. In the particulate filtering efficiency (PFE), aerosols of micro particles, as latex microspheres (PSL), are fired into the mask in order to measure the filtering efficiency. The flow of the aerosol used in the tests is standardized by the international normative and the size of

the particles depends on the size of the microorganism that these particles are representing (generally the BFE test uses particles larger than 300nm and the PFE uses particles smaller than 300nm). The closer the filtering efficiency is to 100% the better, [6], [9], [14], [16], [20].

For the fluid resistance measure, high-speed fluid streams are fired through the mask using human blood pressures of 80, 120 and 160mmHg to simulate blood and other body fluid impact. The fluid resistance measure can be classified in the levels 1, 2 and 3, to denote low, moderate and high risk of fluid exposure, respectively (ASTM F2100-11), [6], [9], [14], [16], [20].

To measure the breathability, the air flow is controlled and the pressure difference is measured over the surface area of the mask to determine its resistance to air flow. The lower the pressure difference the better breathability, [6], [9], [14], [16], [20].

To measure the flammability, masks are set on fire from a specified distance and the time it takes for their flames to spread is measured. The flammability is classified from class 1 to 4, 1 being the best level, [18].

In the case of skin sensitivity, tests are carried out to evaluate the fabric and their constituent materials with regard to their potential to produce irritation and skin sensitization, using ISO 10993-10 as normative. Cytotoxicity is the intrinsic ability of a material to promote metabolic alteration in cells in culture. The tests evaluate the presence and severity of the cytotoxicity of the materials, observing morphological alterations and reduction in the viability of the cells, which may or may not culminate in cell death. Tests use the international guidelines and regulations such as ISO 10993-05, [8], [10], [11].

Due to the great difference between existing materials for homemade masks, there is still no standardization defined specifically for this type of mask. Only BEF, PFE and breathability have been measured for homemade masks, but without conclusions which of them has the best performance, while the other performance measures, although very important, are not investigated in the works in this area. A great difficulty encountered in the analysis of these efficiency measures is the existing conflict of interest between them. For example, a mask with greater coverage, more adjusted to the face and with a great number of layers, improves the filtering efficiency, but worsens other important parameters such thermal comfort, breathability, cost, etc. In this context, the Goal Programming technique is an excellent mathematical tool to assist in choosing the mask by considering simultaneously multiple conflicting factors, [12].

4. Goal Programing

In this research, the Goal Programming (GP) is used for the mathematical modeling of the presented problem. The terminology of Goal Programming was first introduced by [4], although this concept has been presented by [5]. GP is a multiobjective optimization method and thus enables handling of multiple, usually conflicting objectives. In

particular, GP can be successfully used in solving problems in which the task is to find the value of the elements of a variable vector x , where $x = x^* = [x_1^*, x_2^*, \dots, x_k^*]$ which meet L conflicting goals, i.e.:

$$f_j(x^*) = T_j, \quad j = 1, \dots, L, \quad (1)$$

Where variable vector x satisfies the given constraints $x \in F \subseteq R^k$, F is the constraints set of the problem, f_j , $j = 1, \dots, L$ are real functions that describes the objectives and T_j are the targets to be achieved. Since these goals are conflicting, it is impossible to determine x_i^* , $i = 1, 2, \dots, k$, which satisfy (1). Therefore, a relaxation for these goals is considered by creating deviation variables, $p_i \geq 0$ and $n_i \geq 0$, with f_j , $j = 1, \dots, L$, being able to assume values below (n_j) or above (p_j) T_j , that is:

$$f_j(x) - p_j + n_j = T_j, \quad j = 1, \dots, L$$

Where $x \in F \subseteq R^k$ is a vector of variables.

The new objective is to minimize the deviations around T_j . In this way, a function of the unwanted deviation is created according to the characteristics of the problem. Different ways of treating these deviations, produce different variants of Goal Programming. For example:

- In Weighted Goal Programming (WGP) the deviation function is

$$\sum_{j=1}^L u_j n_j + v_j p_j; \text{ the model is}$$

$$\text{Minimize } \sum_{j=1}^L u_j n_j + v_j p_j$$

Subject to:

$$f_j(x) - p_j + n_j = T_j, \quad j = 1, \dots, L$$

$$x \in F, \quad p_j \geq 0 \text{ and } n_j \geq 0, \quad j = 1, \dots, L.$$

In WGP models, weights u_j and v_j are assigned to deviations. Higher weights are assigned to more important goals. The decision-maker plays an important role, as they need to estimate weights according to their preference, [12].

- In Extended Goal Programming (EGP) the deviation function is

$$(1 - \alpha)\gamma + \alpha(\sum_{j=1}^L u_j n_j + v_j p_j); \text{ The adapted model is}$$

$$\text{Minimize } (1 - \alpha)\gamma + \alpha(\sum_{j=1}^L u_j n_j + v_j p_j)$$

Subject to

$$f_j(x) - p_j + n_j = T_j, \quad j = 1, \dots, L$$

$$x \in F, \quad p_j \geq 0 \text{ and } n_j \geq 0, \quad j = 1, \dots, L,$$

where γ is additional objective and $\alpha \in [0,1]$ is a parameter that the Decision-makers need to estimate according to the importance of the objective γ and $(\sum_{j=1}^L u_j n_j + v_j p_j)$. When $\alpha \in (0,1)$ the decision maker can have a trade-off between γ and $(\sum_{j=1}^L u_j n_j + v_j p_j)$. For $\alpha = 0.5$ equal importance is given to both objectives. If $\alpha < 0.5$ more importance is given to objective γ and if $\alpha > 0.5$ the more importance is given to $(\sum_{j=1}^L u_j n_j + v_j p_j)$.

In the traditional EGP, there are constraints $\gamma \geq (u_j n_j + v_j p_j)$, $j = 1, \dots, L$ (γ is defined as an upper limit on the largest deviation), [12]. Although in our model, γ is another objective of the problem, we will still refer to it as adapted EGP.

5. Mathematical Model

In this section, we present a mathematical model to aid the decision making on the best mask to be made. Masks can be made from different types of appropriate materials. The characteristics of each type of mask include the effectiveness of protection it provides against coronavirus, the cost and durability.

The model allows the user to express his/her preferences giving different importance to the economic and performance factors of the mask. A parameter α is introduced, which can take values from $[0, 1]$ interval. When the user wants to give a greater importance to the economic factor, the α value close to 0 should be chosen, while the values close to 1 reflect that the user is more interested in the performance factor. The value 1 for α implies that the user gives importance exclusively to the performance factors of the mask and the model will choose the mask with the best protection against contaminated droplets, independently of the cost. A trade-off between the cost and mask performance could be achieved by giving to α values between 0 and 1.

Let us assume that k masks are evaluated according to L performance measures, which outline the effectiveness of these. Therefore, the E_{ij} value is assigned to each performance measure j of the mask i , $i = 1, \dots, k$, $j = 1, \dots, L$.

The most effective would be the mask z such that $E_{zj} = T_j$, for all j , where T_j is the ideal (target) value for the performance measure j , $j = 1, \dots, L$. However, this equality is utopian, but we can interpret it as a goal to be achieved, and hence the best mask would be the one in which all performance values are as close as possible to the target T_j .

That is,

$$E_{zj} - p_j + n_j = T_j,$$

and all n_j and p_j , $j = 1, \dots, L$ must be non-negative and with values as small as possible. The variables n_j and p_j measure the deviation of E_{zj} from target T_j , $j = 1, \dots, L$; below (n_j) or above (p_j).

Another very important parameters that influence the choice of material for making a mask i is its cost c_i and the durability d_i , that is, the number of times a person can wear the mask i before disposing it, $i = 1, \dots, k$. We introduce the ratio c_i/d_i , $i = 1, \dots, k$ of the cost to the number of times the mask i can be used, $i = 1, \dots, k$.

The decision variables x_i are defined as

$$x_i = \begin{cases} 1, & \text{if the mask of type } i \text{ is chosen for } i = 1, \dots, k \\ 0, & \text{otherwise.} \end{cases}$$

Our optimization model is as follows:

$$\text{Minimize } (1 - \alpha)\gamma + \alpha \left(\sum_{j=1}^L u_j n_j + v_j p_j \right) \quad (2)$$

Subject to

$$\sum_{i=1}^k x_i = 1, \quad (3)$$

$$\sum_{i=1}^k E_{ij} x_i - p_j + n_j = T_j, \quad j = 1, \dots, L, \quad (4)$$

$$\sum_{i=1}^k \frac{c_i}{d_i} x_i \leq \gamma, \quad (5)$$

$$x_i = 0 \text{ or } 1, \quad i = 1, \dots, k, \quad (6)$$

$$\gamma \geq 0, p_j \geq 0 \text{ and } n_j \geq 0, \quad j = 1, \dots, L. \quad (7)$$

where $u_j \geq 0$, $v_j \geq 0$ and α are parameters whose values are to be set by the decision maker, so that $\sum_{j=1}^L u_j + v_j = 1$ and $\alpha \in [0, 1]$. The parameters u_j and v_j can be interpreted as the importance given to each performance measure j , $j = 1, \dots, L$. To give more importance to the performance measure j , the greater value should be given to u_j or v_j associated with the undesired deviations to the target T_j , forcing a greater reduction of the corresponding deviations.

The parameter $\alpha \in [0, 1]$ measures the desired trade-off between the cost and effectiveness of the mask to be chosen. For $\alpha = 0$, the problem comes down to choosing the cheapest mask. For $\alpha = 1$, we seek a mask with the best possible quality, based on the importance given to each performance measure j , $j = 1, \dots, L$. When $\alpha \in (0, 1)$ the decision maker can have a trade-off between the cost and effectiveness of the mask to be chosen. For $\alpha = 0.5$ equal importance to the cost and effectiveness of the mask is given. If $\alpha < 0.5$ more importance is given to the cost and if $\alpha > 0.5$ more importance is given to the effectiveness of the mask to be chosen.

The variable γ , ($\gamma \geq 0$), is an upper limit on cost per use of the mask. To make the sum in the objective function (2) possible the variable γ was created, i.e., it was created to make the first sum portion dimensionless.

We observed that it is possible not to have n_j or p_j for some j . In this case, these variables have the value 0. For example, if j is associated with the PBE efficiency measure, then $p_j = 0$ or we remove p_j from the model as there is no filtering above 100%.

The objective function (2) minimizes the cost per use of the mask and/or the deviations from target T_j . Constraint (3) allows the choice of a single type of mask. Constraints (4) define the goals to be achieved. Constraint (5) (6) and (7) define the variables of the model.

The Model (2)-(7) is formulated as Extended Goal Programming and is a Mixed Integer (binary) Linear Programming problem. As the number of integer variables is not large (it is associated with the number of types of masks) the problem can be solved by any software that implements Mixed Integer Optimization methods. If it is necessary to choose more than one type of mask, the following algorithm can be used.

Algorithm to choose m among k types of masks ($1 \leq m \leq k$):

Algorithm m -Masks

1. *Start*: Given all parameters of the Model (2)-(7).

L = total number of efficiency parameters.

k = total number of types of masks available.

m = number of mask to be chosen.

$\alpha, c_i, d_i, u_j, v_j, E_{ij}$ and T_j are pre-defined, $i = 1, \dots, k$, $j = 1, \dots, L$.

2. *Do*: model = Model (2)-(7).

$y \leftarrow 1$.

3. *While*: $y \leq m$ do

(a) Solve the model.

x^* is the optimal solution, where $x_z^* = 1$ and $x_i^* = 0$, $i = 1, \dots, z-1, z+1, \dots, k$ (The mask z was chosen)

(b) $BestMask(y) = z$.

(c) Include $x_z = 0$ to the constraints set of the *model*.

(d) $y \leftarrow y + 1$.

4. *End-While*

5. *BestMask* contains the list of the best masks in ascending order (the

first one is the best).

6. *End-Algorithm*

6. Computational Experiments

In this section, we present the analysis of the computational experiments that were carried out. The model was solved by Branch and Bound method provided in Solver, which is part of the software LibreOfficeCalc version 6.0 on a computer with Intel Core i5-7500 - 8.0GB RAM. The main aim of the experiments was to analyze the effect of different sets of weights on the selection of the best mask.

The six performance measures chosen to be used in the mask selection process and their ideal values are given in Table 1. Table 2 presents 18 type of materials that can be used to make masks, their performance measures, costs and durabilities. The particle retention efficiency (BEF and PFE) and breathability data take values from 0 to 100%, [14],[20]. The flammability of each material takes integer values from 1 to 4, where the value 1 denotes the least flammable, [18]. The values for the parameter Cost were obtained by taking the average of the 4 values found on commercial websites, calculating the price per unit area of the material (US Dollar per cm^2) and considering a mask with the area of 550 cm^2

(22 cm high per 25 cm wide) per layer. In the case of coffee filter, the cost is proportional to the price of 2 filter units in a box with 80 units. The durability of each mask was determined as the maximum number of times it can be used (washed) in line with the fabric manufacturing companies and ANVISA (Brazilian Health Regulatory Agency) recommendations, [2].

No data on fluid resistance and thermal comfort were found specifically for masks. However their integer values from 1 to 6 were specified based on thermal comfort and humidity tests presented for clothing. The values used in our experiments were given only for the purpose of testing the model and can be easily changed. Based on the discussions and classifications of sensory and thermal comfort clothing, the thermal comfort parameter takes integer values from 1 to 6; the smaller value, the better comfort, [13],[17]. Based on the discussions and classifications of moisture clothing, the fluid resistance takes integer values from 1 to 6; the higher value, the less moisture from liquids in the nose and mouth created by talking and breathing is retained in the mask, [13], [19].

Table 1: Ideal values for the performance measures

j	Performance Measure	Ideal Value for Measure T_i
1	Bacterial filtering efficiency (BEF)	100%
2	Bacterial filtering efficiency (Virus) (PFE)	100%
3	Fluid resistance	6
4	Breathability	2
5	Flammability	1
6	Thermal comfort	1

Table 2: Performance measures, cost and durability for each material

i	Material	Performance Measure j						Cost c_i (dollar)	Durability d_i (days of use)
		1	2	3	4	5	6		
1	cotton quilt	96.1	96.0	4.0	2.7	3	4.0	2.37	35
2	quilter's cotton (80 TPI), 1 layer	14.0	9.0	3.0	2.2	3	1.0	0.39	30
3	quilter's cotton (80 TPI), 2 layers	49.0	38.0	3.5	2.5	3	2.0	0.79	30
4	flannel	44.0	57.0	3.5	2.2	3	4.0	0.18	30
5	cotton (600 TPI), 1 layer	98.4	79.0	4.0	2.5	3	4.0	0.56	35
6	cotton (600 TPI), 2 layers	99.5	82.0	5.0	2.5	3	5.0	1.12	35
7	chiffon, 1 layer	73.0	67.0	4.0	2.7	1	5.0	0.32	20
8	chiffon, 2 layers	90.0	83.0	5.0	3.0	1	6.0	0.63	20
9	natural silk, 1 layer	56.0	54.0	2.0	2.5	1	2.0	0.51	20
10	natural silk, 2 layers	65.0	65.0	2.0	2.7	1	2.5	1.03	20
11	natural silk, 4 layers	88.0	86.0	2.5	2.7	1	3.0	2.05	20
12	cotton/chiffon	99.2	97.0	3.5	3.0	2	4.5	0.71	20
13	cotton/silk	98.5	94.0	2.5	3.0	2	2.5	0.91	20
14	gauze	32.0	37.0	2.5	3.0	2	2.5	0.91	20
15	cotton/flannel	96.0	95.0	3.5	3.0	2	4.5	0.57	30
16	non-woven fabric (80g.m^{-2}), 1 layer	73.0	49.0	6.0	2.1	2	5.0	0.09	1
17	non-woven fabric (80g.m^{-2}), 2 layers	74.0	60.0	6.0	2.2	2	6.0	0.19	1
18	coffee filter paper (100g.m^{-2})	94.0	49.0	6.0	3.5	4	6.0	0.26	1

Source: [2], [13],[14], [17], [18],[19],[20].

The computational experiments were carried out using five different combinations of weights assigned to the ratio of cost to durability and effectiveness of the mask, which illustrate trade-offs that the decision maker may achieve. According to the literature, the most important performance measures to consider when making a mask are Bacterial

filtration efficiency (BEF) ($j = 1$), Particulate filtering efficiency (PFE) ($j = 2$) and Breathability ($j = 4$). Therefore, all experiments in this research consider weights assigned to the performance measures $j = 1, 2$ and only one of them did not consider $j = 4$.

The weights assigned to the performance measures are shown in Tables 4, 6, 8, 10 and 12, while Tables 3, 5, 7, 9 and 11 present the weights u_j and v_j , $j = 1, \dots, 6$ given to each negative and positive deviation from the target value set for each performance measure, respectively. The decision maker can choose the best mask analyzing the values of negative and positive deviations n_j and p_j , $j = 1, \dots, 6$ and the objective function (OF) for each set of weights. Observing Table 1 is noted that we do not need the n_4 , n_5 , n_6 , p_1 , p_2 and p_3 variables, therefore we can remove it from the Model (2)-(7).

Experiment 1: The Model (2)-(7) was applied considering the same weights assigned to all performance measures, as can be seen in Table 3. This experiment simulates a situation in which all efficiency measures are equally important.

Table 3: Experiment 1 - Combination of weights

u_1	u_2	u_3	v_4	v_5	v_6
0.167	0.167	0.167	0.167	0.167	0.167

The results of the proposed methodology using different values for α and equal weights for all performance measures are shown in Table 4. The table shows that the best option is to make mask 4 when the cost of the mask is of only importance ($\alpha = 0$). On the other hand, if the effectiveness of the mask is important only ($\alpha = 1$), the best option is to make mask 11. Furthermore, when the trade-off between the cost and effectiveness is preferred ($\alpha = 0.5$) the best option is to choose mask 13.

Table 4: Experiment 1 - Results achieved using the Model (2)-(7) with the weights shown in Table 3 and different values of α

α	n_1	n_2	n_3	p_4	p_5	p_6	Type i
0	0.560	0.430	0.417	0.050	0.500	0.500	4
0.25	0.040	0.050	0.417	0.250	0.250	0.583	15
0.50	0.015	0.060	0.583	0.250	0.250	0.250	13
0.75	0.015	0.060	0.583	0.250	0.250	0.250	13
1.00	0.120	0.140	0.583	0.175	0	0.333	11

Experiment 2: The Model (2)-(7) was applied considering higher weights for performance measures 1, 2, 4 and 6, and lower weights for performance measures 3 and 5, as can be seen in Table 5. This experiment simulates the most common practical situation in which BEF, PFE, breathability and thermal comfort of the masks made of fabric or paper are considered more important than fluid resistance and flammability.

Table 5: Experiment 2 - Combination of weights

u_1	u_2	u_3	v_4	v_5	v_6
0.2	0.2	0.1	0.2	0.1	0.2

Table 6 shows the results of the proposed methodology using different values for α and weights for everywhere performance measures as shown in Table 3. Mask 4 will be the best choice in all experiments because we only change weights u_j and v_j , $j = 1, \dots, 6$ and they do not matter because $\alpha = 0$. Table 6 shows that the best option is to make mask 8 when the trade-off between the cost and effectiveness is desired. However, when the effectiveness of the mask is important, the best option is to make mask 11.

Table 6: Experiment 2 - Results achieved using the Model (2)-(7) with the weights shown in Table 5 and different values of α

α	n_1	n_2	n_3	p_4	p_5	p_6	Type i
0	0.560	0.430	0.417	0.500	0.500	0.500	4
0.25	0.040	0.050	0.417	0.250	0.250	0.583	15
0.50	0.100	0.170	0.167	0.250	0	0.833	8
0.75	0.120	0.140	0.583	0.175	0	0.333	11
1.00	0.120	0.140	0.583	0.175	0	0.333	11

Experiment 3: The Model (2)-(7) was applied considering higher weights for performance measures 1, 2 and 4, and lower weights for performance measures 3, 5 and 6, as can be seen in Table 7. This experiment simulates a situation addressed in the literature in which BEF, PFE and breathability are the only performance measures considered, [6], [14], [16], [20].

Table 7: Experiment 3 - Combination of weights

u_1	u_2	u_3	v_4	v_5	v_6
0.35	0.35	0	0.3	0	0

Weights assigned to deviations are given in Table 7, while results are presented in Table 8. However, the choice changes this time to mask 1 if the effectiveness measures of the mask are to be considered only, and to mask 12 when the trade-off between the cost and effectiveness is introduced.

Table 8: Experiment 3 - Results achieved using the Model (2)-(7) with the weights shown in Table 7 and different values of α

α	n_1	n_2	n_3	p_4	p_5	p_6	Type i
0	0.560	0.430	0.417	0.050	0.500	0.500	4
0.25	0.040	0.050	0.417	0.250	0.250	0.583	15
0.50	0.008	0.030	0.417	0.250	0.250	0.583	12
0.75	0.008	0.030	0.417	0.250	0.250	0.583	12
1.00	0.039	0.040	0.333	0.175	0.500	0.500	1

Experiment 4: The Model (2)-(7) was applied considering higher weights for performance measures 1 and 2, and lower weights for the other performance measures, as can be seen in Table 9. This experiment simulates other situation addressed in the literature in which BEF and PFE are the only performance measures considered, [9].

Table 9: Experiment 4 - Combination of weights

u_1	u_2	u_3	v_4	v_5	v_6
0.5	0.5	0	0	0	0

The results using the combination of weights given in Table 9 are presented in Table 10. If the equal balance between the cost and effectiveness is preferred the best choice is the mask 12 as well as if only the effectiveness of the mask is desired.

Table 10: Experiment 4 - Results achieved using the Model (2)-(7) with the weights shown in Table 9 and different values of α

α	n_1	n_2	n_3	p_4	p_5	p_6	Type i
0	0.560	0.430	0.417	0.050	0.500	0.500	4
0.25	0.040	0.050	0.417	0.250	0.250	0.583	15
0.50	0.008	0.030	0.417	0.250	0.250	0.583	12
0.75	0.008	0.030	0.417	0.250	0.250	0.583	12
1.00	0.008	0.030	0.417	0.250	0.250	0.583	12

Experiment 5: The Model (2)-(7) was applied considering higher weights for performance measures from 1 to 5, and lower weight for the performance measure 6, as can be seen in Table 11. This experiment simulates a common practical situation in which flammability is not considered for masks made of fabric.

Table 11: Experiment 5 - Combination of weights

u_1	u_2	u_3	v_4	v_5	v_6
0.2	0.2	0.2	0.2	0	0.2

Table 12 shows the results of the proposed methodology using different values for α and weights to performance measures as shown in Table 11. If the cost and effectiveness of the mask are equally important ($\alpha = 0.5$) and if the effectiveness of the mask is important only, the best option is to make mask 8.

Table 12: Experiment 5 - Results achieved using the weights shown in Table 11 and different values of α

α	n_1	n_2	n_3	p_4	p_5	p_6	Type i
0	0.560	0.430	0.417	0.050	0.500	0.500	4
0.25	0.100	0.170	0.167	0.250	0	0.833	8
0.50	0.100	0.170	0.167	0.250	0	0.833	8
0.75	0.100	0.170	0.167	0.250	0	0.833	8
1.00	0.100	0.170	0.167	0.250	0	0.833	8

Experiment 6: The Algorithm m-Masks was applied to determine a list with 6 ($m = 6$) considering only the performance measures 1, 2 and 4 with the same weights, as can be seen in Table 13. This is the scenario presented in [14].

Table 13: Experiment 6 - Combination of weights

u_1	u_2	v_4
0.333	0.333	0.333

Table 14 shows the results of the proposed Algorithm m-Masks using $\alpha = 1$. The results evidence that the 6 best masks, among those studied in [14], are the masks made from cotton and hybrids made from cotton with chiffon, silk and flannel, being cotton quilt the best one in this scenario.

Table 14: List in ascending order of the best masks, where the first one is the best, obtained using Algorithm m-Masks with $m = 6$, $j = 1, 2, 4$ with the weights shown in Table 13 and $\alpha = 1$.

i	Material
1	Cotton quilt
12	cotton/ chiffon
6	cotton (600 TPI), 2 layers
13	cotton/silk
15	cotton/flannel
5	cotton (600 TPI), 1 layer

7. Conclusion

In this paper, we propose a valuable tool to be used when choosing the best material for the mask to be made. This is identified as a very important problem during a pandemic, when there is shortage of medical masks to be offered to public. We used an Operational Research technique, Extended Goal Programming, to aid the decision maker in

selecting the best type of mask to use. In the selection process, the cost and the effectiveness of mask are considered. This problem has conflicting goals and therefore it is not possible to have a single optimal decision. Instead, the proposed model takes into consideration his/her preferences towards the cost and the effectiveness of mask. The decision making tool was developed using a free software tool and we hope this can increase its usability. The current decisions made for choosing homemade masks have been based almost exclusively on particle filtering efficiency, and very important factors such as breathability, thermal comfort and skin sensitivity have not yet been received the necessary importance, which leads people to walk down the street with the mask off the face, or leave the nose free for breathing, or touch the mask to relieve sensations of heat or itching. These attitudes totally weaken (reduce) the performance of the mask and hand hygiene measures. In this sense, the presented methodology is a useful tool to aid in the correct choice of the type of mask to be manufactured.

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